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EXCITATION OF TOLLMIEN–SCHLICHTING WAVES IN THE BOUNDARY LAYER BY THE VIBRATING SURFACE OF AN INFINITE SPAN DELTA WING

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The problem of instability wave origination (Tollmien-Schlichting waves) is discussed extensively at this time in connection with the solution of the problem of predicting the laminar-to-turbulent boundary layer transition point [1, 2]. The problem of exciting Tollmien-Schlichting waves is considered in [3] in the case of a two-dimensional boundary layer on a vibrating surface. This paper is devoted to the solution of the problem [3] in the case of spatial perturbations in the boundary layer in the vibrating surface of an infinite span delta wing.

1. FORMULATION OF THE PROBLEM

Let us consider the flow in the boundary layer on an infinite span delta wing. We select as coordinate system: x is the distance from the leading edge along the streamlined surface, y is the distance along its normal, and the Oz axis is along the wing leading edge. We write the Navier–Stokes equations in dimensionless form by using a certain length scale l, and the free stream velocity U_0. We measure the time in the units l/U_0, the pressure is referred to $\rho_0 U_0^2$ (ρ_0 is the density in the free stream). The temperature and the viscosity coefficient are also measured in units of the corresponding quantities in the free stream. As in [4], we assume that the fundamental flow is weakly inhomogeneous in the absence of perturbations. The following dependence on the coordinates is assumed for the velocity components (U, V, W) and the pressure and temperature (p, T):

$$U = U(x_1, y), \quad V = \varepsilon V_*(x_1, y), \quad W = W(x_1, y),$$
$$p = p(x_1), \quad T = T(x_1, y), \quad x_1 = \varepsilon x, \quad \varepsilon \ll 1. \tag{1.1}$$

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We assume the viscosity coefficient dependent only on the temperature. We write the linearized Navier—Stokes equations with the equation of state taken into account after a Fourier transformation in the time, in the form

$$\frac{\partial}{\partial y}\left(L_0\frac{\partial \mathbf{A}}{\partial y}\right)+L_1\frac{\partial \mathbf{A}}{\partial y}=H_1\mathbf{A}+H_2\frac{\partial \mathbf{A}}{\partial x}+H_3\frac{\partial \mathbf{A}}{\partial z}+\varepsilon H_4\mathbf{A}, \tag{1.2}$$

where L_0, L_1, H_1, H_2, H_3, H_4 are 16 × 16 matrices with L_1 independent of y. All the components containing the derivatives of the fundamental flow functions with respect to x_1 and the velocity component V_* from (1.1) are isolated in the matrix H_4. The vector function A in (1.2) is defined as follows in terms of the perturbation: A_1 is the x component of the velocity; $A_2 = \partial A_1/\partial y$; A_3 is the y component of the velocity, A_4 is the pressure, A_5 is the temperature, $A_6 = \partial A_5/\partial y$; A_7 is the z component of the velocity, $A_8=\partial A_7/\partial y$; $A_9=\partial A_1/\partial x$; $A_{10}=\partial A_3/\partial x$; $A_{11}=\partial A_5/\partial x$; $A_{12}=\partial A_7/\partial x$; $A_{13}=\partial A_1/\partial z$; $A_{14}=\partial A_3/\partial z$; $A_{15}=\partial A_5/\partial z$; $A_{16}=\partial A_7/\partial z$. We assume that initial data are given in a certain section $x = x_0$ in the form of the vector functions

$$\mathbf{A}(x_0,\ y,\ z)=\mathbf{A}_0(y)\exp(i\beta z). \tag{1.3}$$

We simulate the vibration of the streamlined surface on the section being analyzed as a small-amplitude traveling wave. For the considered Fourier harmonic in time with the frequency ω we represent the equation of the surface y(x, z) in the form

$$y=a\exp\left[i\alpha_0(x-x_0)+i\beta z\right].$$

Analogously to [3], we arrive at the following boundary conditions for y = 0:

$$\begin{gathered}A_1(x,0,z)=-aU_w'\exp\left[i\alpha_0(x-x_0)+i\beta z\right],\\ A_3(x,0,z)=-i\omega a\exp\left[i\alpha_0(x-x_0)+i\beta z\right],\\ A_5(x,0,z)=0,\quad A_7(x,0,z)=-aW_w'\exp\left[i\alpha_0(x-x_0)+i\beta z\right],\end{gathered} \tag{1.4}$$

where U_W', W_W' are the values of the derivatives of U and W with respect to y, evaluated at y = 0. The terms $O(a^2)$ are discarded in (1.4). Boundedness of the solution is assumed as $y \to \infty$:

$$|A_j|<\infty\quad(j=1,\ldots,16). \tag{1.5}$$

The problem (1.2)-(1.5) is incorrect. Hence, we impose the condition on the initial data that they allow a solution with a finite index of growth [3].

2. BIORTHOGONAL VECTOR SYSTEM

The solution of the problem (1.2)-(1.5) for the case when the fundamental flow is weakly inhomogeneous in the coordinate x is represented in the form of an expansion in a biorthogonal system of vectors of the locally homogeneous problem $\{\mathbf{A}_{\alpha\beta}(x_1, y), \mathbf{B}_{\alpha\beta}(x_1, y)\}$ [3]. The general principles for the construction of a biorthogonal system for three-dimensional boundary layers are formulated in [1]. Given below are specific equations [1]:

$$\begin{gathered}\frac{\partial}{\partial y}\left(L_0\frac{\partial \mathbf{A}_{\alpha\beta}}{\partial y}\right)+L_1\frac{\partial \mathbf{A}_{\alpha\beta}}{\partial y}=H_1\mathbf{A}_{\alpha\beta}+i\alpha H_2\mathbf{A}_{\alpha\beta}+i\beta H_3\mathbf{A}_{\alpha\beta},\\ A_{\alpha\beta1}=A_{\alpha\beta3}=A_{\alpha\beta5}=A_{\alpha\beta7}=0\ \text{ for }\ y=0,\\ |A_{\alpha\beta j}|<\infty\ \text{ for }\ y\to\infty\ (j=1,\ldots,16);\end{gathered} \tag{2.1}$$

$$\begin{gathered}\frac{\partial}{\partial y}\left(L_0^*\frac{\partial \mathbf{B}_{\alpha\beta}}{\partial y}\right)-L_1^*\frac{\partial \mathbf{B}_{\alpha\beta}}{\partial y}=H_1^*\mathbf{B}_{\alpha\beta}-i\bar{\alpha}H_2^*\mathbf{B}_{\alpha\beta}-i\bar{\beta}H_3^*\mathbf{B}_{\alpha\beta},\\ B_{\alpha\beta2}=B_{\alpha\beta4}=B_{\alpha\beta6}=B_{\alpha\beta8}=0\ \text{ for }\ y=0,\\ |B_{\alpha\beta j}|<\infty\ \text{ for }\ y\to\infty\ (j=1,\ldots,16),\end{gathered} \tag{2.2}$$

where the asterisk * denotes the conjugate matrix, the upper bar denotes the complex conjugate, and the subscripts α and β denote whether the vector functions belong to the solution of problems (2.1) and (2.2) for given parameters α_1 and β. The systems (2.1) and (2.2) depend on the "slow" coordinate x_1 as on a parameter. The equations for the first eight components in (2.1) and (2.2) are split off. They define the rest uniquely. For given values of the frequency ω and the parameter β the problems (2.1) and (2.2) have a discrete and continuous spectrum of allowable values of the parameter α. This analysis is analogous to [5] for two-dimensional boundary layers. The three-dimensionality of the fundamental flow and the perturbations does not yield any distinctions, in principle, from [5]. The following orthogonality conditions hold [1]:

$$\langle H_2\mathbf{A}_{\alpha\beta}, \mathbf{B}_{\gamma\beta}\rangle = \Delta_{\alpha\gamma}, \quad \langle \mathbf{A}, \mathbf{B}\rangle = \lim_{\tau\to 0}\sum_{j=1}^{16}\int_0^\infty e^{-\tau y}A_j\overline{B}_j dy,$$

where $\tau > 0$; $\Delta_{\alpha\gamma}$ is the Kronecker symbol if one of the numbers α, γ is referred to the discrete spectrum, $\Delta_{\alpha\gamma} = \delta(\alpha - \gamma)$ is the delta function if both numbers α, γ are referred to the continuous spectrum.

If $z(x_1, y)$ denotes a vector consisting of the first eight components of the vector $A_{\alpha\beta}$, then the problem (2.1) can be reduced to a well-known system of the Lees-Lin type [4]:

$$dz/dy = H_0 z,\ z_1 = z_3 = z_5 = z_7 = 0 \text{ for } y = 0, \quad |z_j| < \infty \text{ for } y \to \infty\ (j = 1, \ldots, 8), \tag{2.3}$$

where H_0 is a 8×8 matrix. The specific form of H_0 is presented in [4], for example. The system (2.3) has eight linearly independent solutions. Setting the derivatives of the fundamental flow functions with respect to the coordinate y equal to zero outside the boundary layer, we obtain a system of ordinary differential equations with constant coefficients [6]. Seeking its solution $\sim\exp(\lambda y)$, we obtain the characteristic equation for λ:

$$(b_{11} - \lambda^2)^2[(b_{22} - \lambda^2)(b_{33} - \lambda^2) - b_{23}b_{32}] = 0,$$
$$b_{11} = H_0^{21}, \quad b_{22} = H_0^{42}H_0^{24} + H_0^{43}H_0^{34} + H_0^{46}H_0^{64} + H_0^{48}H_0^{84}, \tag{2.4}$$
$$b_{23} = H_0^{42}H_0^{25} + H_0^{43}H_0^{35} + H_0^{46}H_0^{65} + H_0^{48}H_0^{85}, \quad b_{32} = H_0^{64}, \quad b_{33} = H_0^{65},$$

where H_0^{ij} are elements ofthe matrix H_0 evaluated outside the boundary layer. Equation (2.4) has two doubly degenerate roots $\lambda_1 = \sqrt{b_{11}}, \lambda_2 = -\sqrt{b_{11}}$. We denote the two linearly independent vectors corresponding to λ_1 by V_1 and V_7. Their components different from zero are

$$V_{11} = e^{\lambda_1 y}, \quad V_{21} = \lambda_1 e^{\lambda_1 y}, \quad V_{31} = (H_0^{37}/\lambda_1)e^{\lambda_1 y}, \quad V_{37} = (H_0^{37}/\lambda_1)e^{\lambda_1 y}, \quad V_{77} = e^{\lambda_1 y}, \quad V_{87} = \lambda_1 e^{\lambda_1 y},$$

where V_{ij} denotes the i-th component of the j-th vector. We denote the two linearly independent solutions corresponding to λ_2 by V_2 and V_8. Moreover, (2.4) has two roots λ_3, λ_4:

$$\lambda_{3,4} = \pm\{(1/2)(b_{22} + b_{33}) + \sqrt{(1/4)(b_{22} - b_{33})^2 + b_{23}b_{32}}\}^{1/2}.$$

We denote their corresponding linearly independent solutions by V_3 and V_4. The remaining roots λ_5, λ_6 are determined by the inequality

$$\lambda_{5,6} = \pm\{(1/2)(b_{22} + b_{33}) - \sqrt{(1/4)(b_{22} - b_{33})^2 + b_{23}b_{32}}\}^{1/2}.$$

The linearly independent solutions V_5 and V_6 correspond to them. For definiteness we select the branches $\mathrm{Real}\,\lambda_1 < 0$, $\mathrm{Real}\,\lambda_3 < 0$, $\mathrm{Real}\,\lambda_5 < 0$. Tollmien–Schlichting waves correspond to solutions of the discrete spectrum. We denote the solution of (2.3) for them by z_{TS}:

$$\mathbf{z}_{TS} = c_1\mathbf{V}_1 + c_3\mathbf{V}_3 + c_5\mathbf{V}_5 + c_7\mathbf{V}_7. \tag{2.5}$$

One of the coefficients in (2.5) is arbitrary because of the linearity of the problem. The rest are determined from the boundary conditions for y = 0. Here α_{TS} is determined from the dispersion relationship (the subscript TS denotes belonging to the discrete spectrum):

$$E_{1357}(\alpha_{TS}) = \det\begin{vmatrix} V_{11} & V_{13} & V_{15} & V_{17} \\ V_{31} & V_{33} & V_{35} & V_{37} \\ V_{51} & V_{53} & V_{55} & V_{57} \\ V_{71} & V_{73} & V_{75} & V_{77} \end{vmatrix}_{y=0} = 0.$$

To construct the solution of the problem (1.2)-(1.5) in the form of an expansion in the eigenvectors $A_{\alpha\beta}$ later, we construct the vector $A_v(x_1, y)$ analogously to [3]:

$$\frac{\partial}{\partial y}\left(L_0\frac{\partial \mathbf{A}_v}{\partial y}\right) + L_1\frac{\partial \mathbf{A}_v}{\partial y} = H_1\mathbf{A}_v + i\alpha_0 H_2\mathbf{A}_v + i\beta H_3\mathbf{A}_v,$$
$$A_{v1} = -aU_w', \quad A_{v3} = -ia\omega, \quad A_{v5} = 0, \quad A_{v7} = -aW_w' \text{ for } y = 0,$$

$$|A_{vj}| \to 0 \text{ for } y \to \infty \; (j = 1, \ldots, 16).$$

We let z_0 denote a vector consisting of the first eight components of A_v. It can be written in the form

$$z_v = a(d_1 V_1 + d_3 V_3 + d_5 V_5 + d_7 V_7)/E_{1357}(\alpha_0),$$

where the coefficients d_j are determined from the boundary conditions for $y = 0$. The vector z_v depends on x_1 as on a parameter. We note that if there is a resonance point $x_1 = x_*$ at which $\alpha_{TS} = \alpha_0$, then z_v has a pole.

3. GENERATION OF TOLLMIEN-SCHLICHTING WAVES

We seek the solution of the problem (1.2)-(1.5) in the form

$$\mathbf{A}(x, y, z) = \sum{}' c_\alpha (x_1) \mathbf{A}_{\alpha\beta}(x_1, y) \exp\left\{i \int_{x_0}^{x} \alpha dx + i\beta z\right\} + \mathbf{A}_v(x_1, y) \exp\{i\alpha_0(x - x_0) + i\beta z\}, \tag{3.1}$$

where the $\sum'$ denotes summation over the discrete and integration over the continuous spectrum. Limiting ourselves to the examination of only components with A_v and A_{TS} in (3.1) and repeating the calculations [3], we find the coefficient $c_{TS}(x_1)$ and we see that the solution (3.1) is uniformly suitable in x. Using the saddle-point method [7] here, we find the amplitude of the Tollmien-Schlichting wave excited in the neighborhood of the resonance point $x_1 = x_*$, where $\alpha_{TS}(x_*) = \alpha_0$. If we are interested in a specific physical quantity q (the amplitude of the fluctuations in velocity, temperature, or mass flow rate, etc.) in the excited wave, then its value c_q has the form:

$$\frac{|c_q|}{a} = \frac{1}{\sqrt{\varepsilon}} \sqrt{2\pi \left| \frac{d\alpha_{TS}(x_*)}{dx_1} \right|} \, q / \left| \langle H_2 \mathbf{A}_{TS}, \mathbf{B}_{TS} \rangle_{x_1 = x_*} \right|, \tag{3.2}$$

where the quantity q is determined in terms of the components of the vector A_{TS}. The vectors A_{TS}, B_{TS} in (3.2) are determined from (2.1) and (2.2) for β from (1.3). It can be shown by quite tedious calculations that

$$\langle H_2 \mathbf{A}_{TS}, \mathbf{B}_{TS} \rangle = -i \left\langle \frac{\partial H_0}{\partial \alpha_{TS}} \mathbf{z}_{TS}, \chi_{TS} \right\rangle + O(\mathrm{Re}^{-1}),$$

where Re is the Reynolds number and χ_{TS} is the solution of the adjoint problem

$$d\chi/dy = -H_0^* \chi, \; \chi_2 = \chi_4 = \chi_6 = \chi_8 = 0 \text{ for } y = 0, \quad |\chi_j| \to 0 \text{ for } y \to \infty \; (j = 1, \ldots, 8), \; \alpha = \alpha_{TS}. \tag{3.3}$$

4. EXAMPLE OF A NUMERICAL COMPUTATION

Considered as an illustration in this paper is the symmetric profile NACA 0012 at zero angle of attack, for which the sweepback angle $\psi_c = 30°$ is given. The chord length was selected at 1.5 m, the free stream pressure and temperature were 10^4 N/m^2 and 300°K, respectively, and the Mach number was M = 0.28. The coefficient of viscosity was assumed dependent on the temperature according to the Sutherland formula. The Prandtl number was 0.72. The boundary layer calculation was excuted within the framework of a locally self-similar approximation [8]. Linearly independent solutions for systems of differential equations (2.3) and (3.3) were found numerically by using an orthogonalization method [9, 6]. The dependence of the amplitudes of the maximal value of the mass flow rate for the x and z components of the Tollmien-Schlichting waves (curves 1 and 2, respectively) is represented in Fig. 1 as a function of the angle $\psi = \mathrm{arctg}\,(\beta/\alpha)$ in the case of a resonance excitation regime for a 500-Hz pertubation frequency. Numerical values of the amplitude of the surface vibrations are presented in dimensional form per 1 m. From the results presented in the figure, there follows that the value ~10^{-6} m of the vibration amplitude yields a ~1% fluctuation amplitude in the unstable zone in the case of the resonance regime of Tollmien-Schlichting wave excitation.

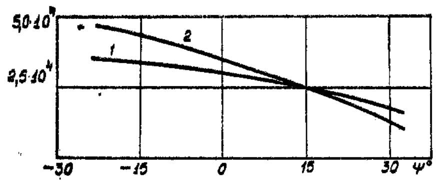

Fig. 1

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STRUCTURES AND THEIR EVOLUTION IN A TURBULENT SHEAR LAYER

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1. INTRODUCTION

From the mathematical viewpoint, turbulent fluid motion is represented by the result of exciting many strongly interacting degrees of freedom. In the motion of these degrees of freedom there is hence neither total chaos (which would permit utilization of simple statistical models), nor total coherence. Recent investigations (see e.g., [1-3]) make the idea that many turbulent flows are a system of interacting and quite stable wave packets, vortex structures, all the more likely. The spatial separateness often observed for the structures indicates that their interaction does not annihilate the possibility of considering a structure as a certain "unit" of turbulence.

There is apparently no single mechanism for the formation of structures in different turbulent flows. The widely known dissipative structures are represented by the combined product of nonlinearity and dissipation. For instance, Benard cells in convective flows and Taylor vortices in circular Couette flows originate and exist in a limited range of nonlinearity-to-dissipation ratios. In free turbulent flows, jets, wakes, and in mixing layers the dissipation plays no visible part in structure formation. It can be assumed that certain local integrals of motion are responsible for the existence of structures in these effectively nonviscous flows. The prolonged existence of structures naturally results in the idea of building up an internal statistical equilibrium therein [4-6]. As has been shown in [7, 8], isolated statistically equilibrium structures from two-dimensional point vortices

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